Phase Measurement System for Inter-Spacecraft Laser Metrology

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Abstract— **Radio ranging signals have been used extensively for mapping the Earth's gravity field. The latest example is the GRACE (Gravity Recovery and Climate Experiment) mission which uses centimeter-wavelength radio signals to measure changes in distance between two spacecraft with sub-micron accuracy. Range change is determined by measuring changes in the phase of the received radio signal. Improved range accuracy can be achieved by using lasers with one micron wavelengths for the ranging signal. The laser phase measurement electronics must cope with higher signal dynamics due to Doppler shifts and more noise in the measured phase (which, multiplied by the smaller wavelength, still results in a more accurate ranging measurement). A new phase measurement system has been developed and tested to meet these requirements. It is based on the digital signal processing system used for GPS receivers and the GRACE mission. High-speed digitizers sample the laser phase signal, which is digitally filtered in a field-programmable gate array. The resulting phase measurement system is capable of measuring the phase of subcarrier frequencies up to 20 MHz and frequency rate up to 100 Hz/s with microcycle accuracy at mHz signal frequencies in the presence of laser noise. The system also includes output of the phase measurement at a rate of 1 MHz which can be used in locking one laser transmitter to another. The phase measurement system may also have other applications with demanding measurement requirements, including on the LISA gravitational-wave mission.**

Index Terms— **Phasemeter, laser ranging, gravity wave, LISA, radio ranging, gravity field mapping, GRACE, GPS**

I. Introduction

The gravity field of the Earth, planets, and solar system have been best measured by determinating the precise trajectories of orbiting spacecraft. Measurements of distance or velocity from an Earth tracking station or Earth orbiting (i.e. Global Positioning System) satellite are some of the most precise means of trajectory determination. One of these methods involves the measurement of the phase of an electromagnetic signal propagating between a transmitter and the satellite. Measurement of the phase provides a measure of range to a fraction of the wavelength of the signal. The range determined in this way is very precise but is subject to an ambiguity in the integer number of cycles between the transmitter and receiver. This ambiguity can sometimes be resolved, for example by measurement of the transmitted signal by multiple receivers or by modulating the signal with a ranging signal of suitable bandwidth. Often, however, it is the change in range over time that is most significant and the ambiguity can be ignored or estimated as a bias parameter as part of the orbit estimation.

The GRACE (Gravity Recovery and Climate Experiment) mission [1] currently in orbit is providing the best gravity field determinations yet achieved. GRACE includes two spacecraft measuring the changes in distance between them by measuring the phase of a microwave signal with wavelength of about 1 cm. With measurements of the change in distance to about one part in ten thousand, the GRACE mission is able to measure changes in mass distribution on the Earth due to effects such as melting of the Antarctic ice cap.

A future Earth gravity mapping mission may be able to achieve much higher accuracy of gravity field determination, allowing measurements of mass change due to, for example, changes in local water tables, at much better spatial resolution than GRACE has achieved. One of the limiting error sources for the GRACE measurement is the accuracy of the inter-spacecraft range measurement. By using a laser signal rather than a microwave signal the range measurement accuracy can be improved both by use of a shorter wavelength and by achieving better signal-to-noise ratio made easier by the lower diffractive losses in the shorter wavelength system.

A similar inter-spacecraft laser ranging system is envisioned for the LISA gravitational-wave observatory [2]. LISA is designed to measure changes in distance due to gravitational waves generated outside the solar system, rather than local mass distribution changes. LISA involves three spacecraft separated by 5 million kilometers orbiting about the sun.

Because laser signals have shorter wavelengths than microwave signals, the change in frequency due to motion of the spacecraft is much larger. The phase measurement electronics need to be able to cope with the resulting larger dynamic range. For GRACE the microwave signals transmitted between spacecraft are referenced to stable crystal oscillators on each spacecraft. The phase of the received signal is measured separately on each spacecraft against the local crystal oscillator. The phase measurements are combined numerically on the ground to remove the oscillator effects and determine the range. Rather than lock lasers to independent references on each spacecraft, it is planned to use the laser on one spacecraft as the reference and to lock lasers on the other spacecraft to the received signal from the reference laser.

We describe below the development of phase measurement electronics with these missions in mind. The measurement electronics are capable of handling the high dynamic range required for the laser ranging system with the high accuracy required for both a future Earth gravity mapping mission and for a gravitational wave observatory.

II. Phasemeter requirements

The phase measurement electronics are intended to perform two functions. The first is the measurement of the phase of the input signal for the science measurement which is sent as telemetry to the ground. The phase measurement accuracy needed is about 100 $\mu\text{cycles}/\sqrt{\text{Hz}}$ for a future Earth gravity mapping mission. This is the same phase measurement accuracy as for GRACE with much better resultant range accuracy due to the shorter wavelength. For LISA the needed measurement accuracy is about 5 $\mu\text{cycles}/\sqrt{\text{Hz}}$. The second function is to determine the phase difference between two laser frequencies and to provide that signal in real time at high bandwidth so that one laser may be locked to the other. For both functions the signal whose phase is to be measured is the output of a photodiode on which the beams from two lasers are interfered.

The dynamic range of the phase measurement is affected both by the orbital dynamics of the spacecraft and by the short-term fluctuations in frequency of the lasers.

For a gravity mapping mission, the desired orbits are low in altitude, to get the strongest possible gravity signature, and high inclination, to enable measurement of the gravity field at all latitudes. For two spacecraft in low circular nearly polar orbits with spacecraft separation of about 100 km, the oblateness of the Earth causes a periodic relative velocity of about ± 1 m/s. For a laser wavelength of nominally 1μm, this results in a frequency variation of ± 1 MHz with a period of about 90 minutes giving a maximum frequency rate of 2.5 kHz/s. For the LISA mission, the relative motion between two spacecraft is higher, up to ± 20 m/s, but with a longer 1 year period.

The reference laser will be stabilized to either a thermally isolated reference cavity or to an atomic line transition. The laser frequency noise is expected to be about 30 $\text{Hz}/\sqrt{\text{Hz}}$ at 1 Hz, or about one part in 10^{13} for a root-mean-square deviation over 1 second averaging times. This is about the same fractional stability as for microwave system but much larger in terms of phase due to the shorter wavelength/higher frequency of the laser signal. At the frequencies of interest for gravity mapping and observation of gravitational waves (0.001 Hz to 1 Hz), the laser frequency noise corresponds to a phase noise of 5000 $\text{cycles}/\sqrt{\text{Hz}}$ at 1 mHz. To obtain the desired accuracy of 5 $\mu\text{cycles}/\sqrt{\text{Hz}}$, the phasemeter must have a dynamic range of 10^9.

In addition, some of these applications require that the laser have several additional tones for inter-spacecraft communication, clock noise removal, etc., so the phasemeter must be able to measure multiple tones.

III. Phasemeter architecture

Several different types of phase measurement were considered. One common technique to measure phase is to accurately time the zero-crossing points of the signal waveform. This approach suffers from poor performance in the presence of broadband noise due to aliasing of noise at harmonics of the heterodyne signal. The phasemeter developed here is based on a digital phase-locked loop (DPLL) technique. The architecture used is shown in Figure 1.

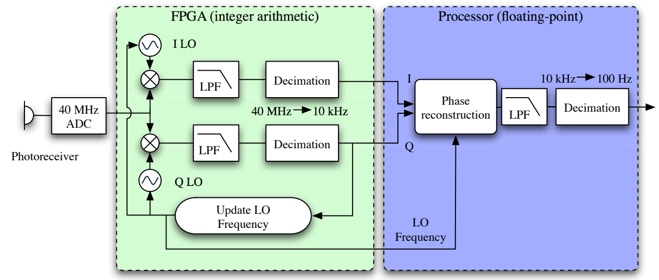


Fig. 1. Phasemeter block diagram, showing the two stage integer floating point gate array (FPGA), and the floating point real-time processor. The 40 MHz digitized signal is filtered and decimated to 10 kHz in the FPGA, then filtered and decimated again to 100 Hz for transmission to ground. The signal bandwidth for these observations is from 1 mHz to 1 Hz. LO: local oscillator. LPF: low pass filter. ADC: analog to digital converter.

The output of the photoreceiver is digitized at 40 MHz, then fed into a field programmable gate array (FPGA). The signal is then multiplied by a local oscillator (LO) at the same frequency as the signal. In stable operation, the output of this multiplier is proportional to the phase difference between the LO and the signal. This phase difference is then used to update the LO frequency (and thereby phase) to keep the DPLL locked to the incoming signal.

The phase of the local oscillator is approximately equal to the phase of the signal. Imperfections in locking will limit the accuracy of the phase estimate based on such a system. The signal phase can be recovered by measuring the residual tracking error and combining this with the LO phase.

$$\phi_{\text{signal}} = \phi_{\text{LO}} + \phi_{\text{residual}} \tag{1}$$

The residual phase is measured using in-phase and quadrature demodulation as described in section III-B.

A. Analog to digital conversion

Quantization error of the analog to digital converter (ADC) will produce phase noise. The root power spectral density (RPSD) phase noise as a function of sampling rate, f_s and number of bits N is

$$\tilde{\phi} = \frac{1}{\pi 2^N \sqrt{6 f_s}}. \tag{2}$$

With a 15 bit ADC sampling at 40 MHz, this quantization noise is negligible.

Another source of error introduced by the ADC is jitter in the sampling time. The RPSD phase noise as a function of timing jitter RPSD $\tilde{t_s}$ and heterodyne frequency f_h is

$$\tilde{\phi} = \frac{\tilde{t_s}}{f_h}. \tag{3}$$

For the phasemeter demonstrated here, the sampling time jitter is less than 10 $\mu\text{cycles}/\sqrt{\text{Hz}}$ with a 5 MHz heterodyne frequency (Figure 8.

B. Digital demodulation and phase-locked loop

The dopper shift between the outgoing and incoming lasers results in a heterodyne frequency ω which can be decomposed into the in-phase I and quadrature Q components of the signal

$$A(t)\sin(\omega t+\phi(t)) = I(t)\cos\omega t + Q(t)\sin\omega t \tag{4}$$

where

$$I(t) = A(t)\cos\phi(t) \tag{5}$$

$$Q(t) = A(t)\sin\phi(t) \tag{6}$$

and $\phi(t)$ is the quantity which we wish to measure. The in-phase and quadrature components can be extracted by multiplying the signal by cosine and sine of the local oscillator phase.

The frequency of the local oscillator is determined by a tracking loop which feeds back the quadrature signal Q to change the frequency of the LO. If the error between the incoming frequency and the LO frequency is small enough, i.e., the loop is tracking well, $Q \propto \sin\phi(t) \simeq \phi(t)$ is proportional to the error in the phase. The tracking loop operates at the 10 kHz output frequency, so the LO frequency is updated every 0.1 ms.

C. Precision filtering and decimation

The signal is sampled at 40 MHz. For most purposes, however, the phase measurements are needed at a lower data rate, so the high rate measurements are decimated. One potential source of error in the phasemeter processing is aliasing of noise during this decimation, so digital anti-aliasing filters are needed. The requirements of the anti-aliasing filter depend upon the characteristics of the signal phase noise.

Any anti-aliasing filter must be phase stable and linear in the passband, or it will distort the phase measurement. The signal band is small compared with the heterodyne frequency, so we can allow aliasing everywhere but in the signal band from 1 mHz to 1 Hz. For example, when decimating from 10 kHz to 100 Hz, only noise near 10 kHz and harmonics will alias into the signal band. By carefully placing the nulls of the filter at nf_s, we can achieve very good rejection of aliasing in the signal band, as shown in Figure 2.

To implement the filter in the integer FPGA, we used a Bartlett triangular filter. This filter can be efficiently implemented by convolving two top-hat functions. As the filter is immediately followed by a decimation stage, we can save computation by only calculating the points that will be retained, reducing the number of calculations needed by the ratio of the output rate to the input rate. Figure 3 shows the behavior of the filter at the nulls and its quadratic response, allowing excellent anti-aliasing in the desired signal band.

D. Phase reconstruction

The LO frequency estimate feeds back to the local oscillator demodulation to maintain the phase-locked loop, and feeds forward to the phase reconstruction. The LO frequency could simply be integrated to obtain the phase (ϕ_{LO} in Equation 1), but to achieve microcycle accuracy, the residual I and Q are also used in the phase reconstruction. With the demodulated and filtered I and Q, the residual phase ϕ_{residual} of Equation 1 can be calculated with a four-quandrant arctangent[1]. A subtle complication is that the Bartlett filter takes two 10 kHz cycles to get one 10 kHz model frequency point, while I and Q are calculated for each cycle, so the residuals must be carefully averaged to achieve microcycle accuracy.

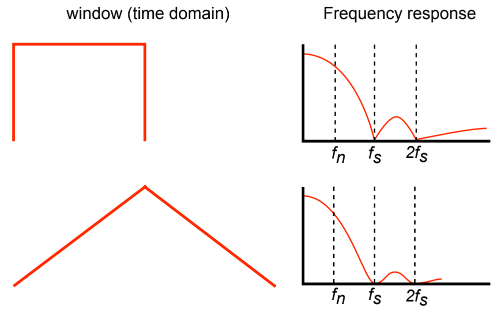


Fig. 2. Schematic representation of top hat window versus Bartlett in the frequency domain. While any unfiltered noise above the Nyquist frequency f_n will alias, only noise at multiples of nf_s will alias back into the signal band from 1 mHz to 1 Hz. All noise at frequencies above the signal band will be rejected in the final filter/decimation stage.

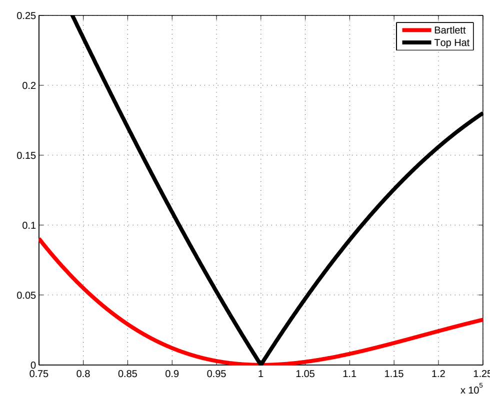


Fig. 3. Expanded view of top hat window versus Bartlett at the nulls.

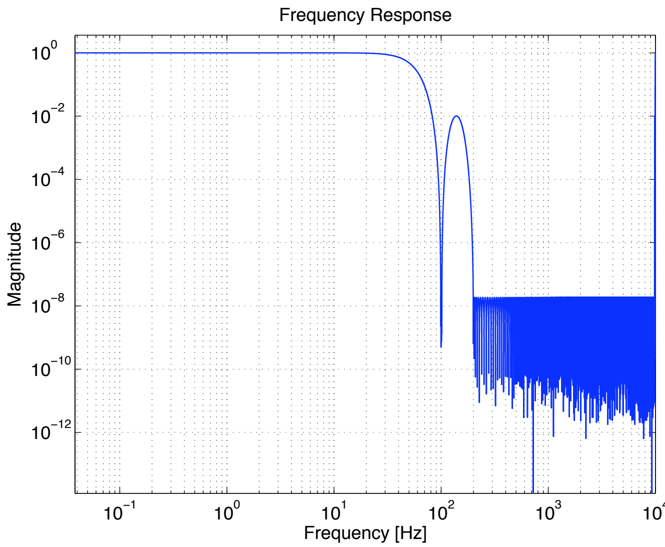
E. Decimation to final output

The signal sent to ground must be at a much lower rate than 10 kHz. After the phase reconstruction at 10 kHz, a finite impulse response (FIR) filter is used prior to decimation to the final rate of 100 Hz. To minimize computation we use the same principle of only calculating the decimated points that we are going to keep. A 600 point FIR filter kernel was designed to be flat in the passband to one part in 10^7 and have a rejection of one part in 10^8 in the relevant parts of the stop band (Figure 4).

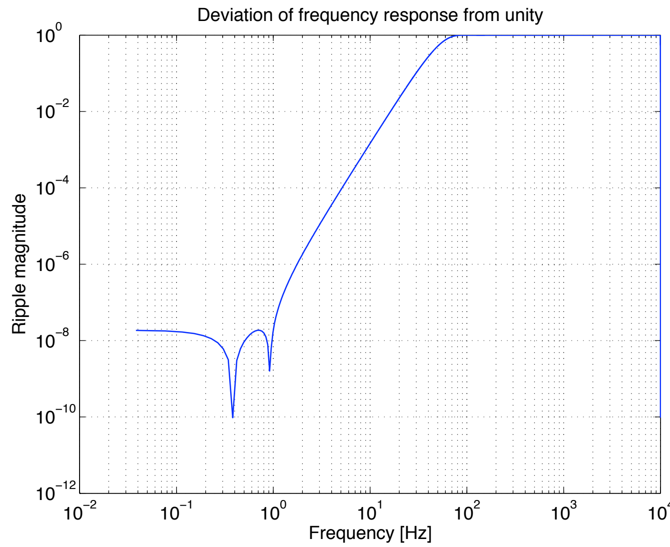
F. Dithering

The quantization noise described in Equation 2 applies when the quantization error is smaller than the other noise

[1] Arctangent involves a division, which is difficult to implement on an integer FPGA. We have implemented an all-FPGA integer arithmetic version of Figure 1, but it requires either a large amount of real-estate or a large amount of memory for a look-up table, and so has fewer phasemeter channels.



(a) Frequency response of decimation filter



(b) Deviation of filter from unity in the passband

Fig. 4. 10 kHz to 100 Hz decimation filter

on the signal and when the signal frequency divided by the sampling frequency is not a rational number. Since we can neither prevent the signal frequency from sweeping through frequencies which are rational divisors of the clock, nor ensure the amplitude will not be exactly between two bit transitions, we add a small amount of noise in the least significant bit. This gains the benefits of averaging and ensures the noise characteristics given by Equation 2. To generate the noise, we developed a pseudo-random number generator based on a linear shift feedback register, and used a triangular distribution to dither the least significant bit. The FPGA-generated pseudo-random number generator has proven to be quite useful for many applications, e.g., generating correlated noise for testing and shot noise for simulation.

G. Acquisition and gain control

The FPGA phasemeter also includes an initial frequency estimation stage based on counting the zero crossings of the signal, and an automatic gain control which multiplies the signal up to maximize the number of bits used in the calculation of the I and Q demodulation. Since the number of bits needed to obtain the desired microcycle accuracy is much less than the number of bits given by the chosen ADC, the system is robust to large amplitude changes.

H. Implementation

The phasemeter is implemented on a single field programmable gate array. The FPGA is programmed using LabVIEW, which is also used as the interface to the user. The ADC is based on a Maxim development board, which digitizes 15 bits at 40 MHz. A separate version of the phasemeter has been developed for phase-locking one laser to another and uses a 250 kHz digital-to-analog converter to drive the temperature and piezo-electric frequency actuator of the Nd:YAG laser to maintain a microcycle stable phase lock from 1 mHz to 1 Hz.

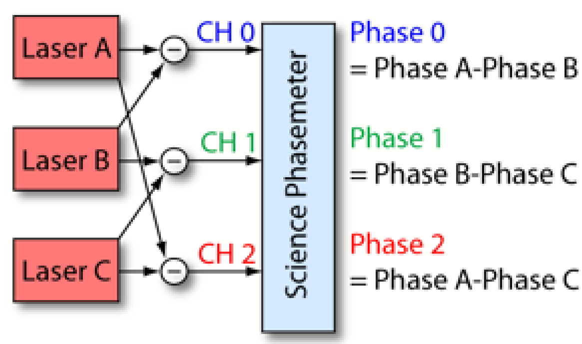


Fig. 5. Block diagram of phasemeter linearity test

IV. RESULTS

Two complementary tests were done to show that the phasemeter and ADC meet the requirements. Figure 5 is a block diagram of a linearity test, simulating three individual lasers which are phase-locked to each other. The diffence between each pair of lasers looks like uncorrelated phase noise, as shown in Figure 6.

When the three phase difference outputs are linearly combined appropriately, the noise should add to zero; any residual noise is due to non-ideal phasemeter and front-end electronics behavior. Figure 7 shows this to be at the microcycle level. These are the results for a test where the noise signals were generated digitally, bypassing the ADCs. The root power spectral density of the noise for the sum of the signals is less than 5 μcycles/$\sqrt{\text{Hz}}$, showing that the numerical part of the phasemeter easily meets the requirements.

Figure 8 shows the phasemeter response with the same 4 MHz input on different ADC channels. The difference between the two channels is below 10 μcycles down to 1 mHz, which meets requirements. The phase error rising inversely as frequency on Phase 0 and Phase 1 is due to the noise on the

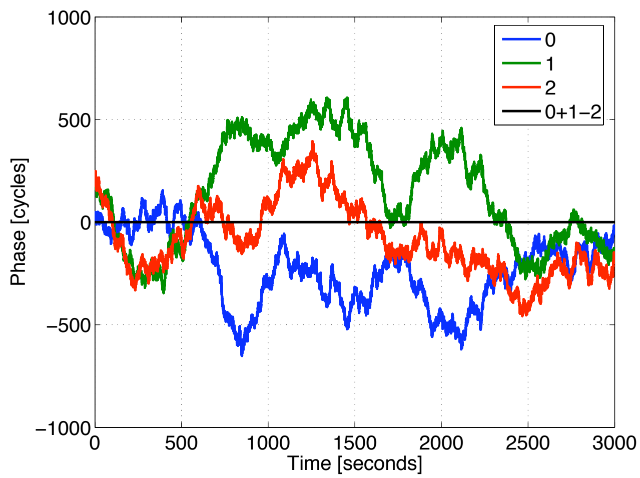


Fig. 6. Sample time series showing the phasemeter measurement from three correlated lasers, and the result of adding the three phase outputs correctly.

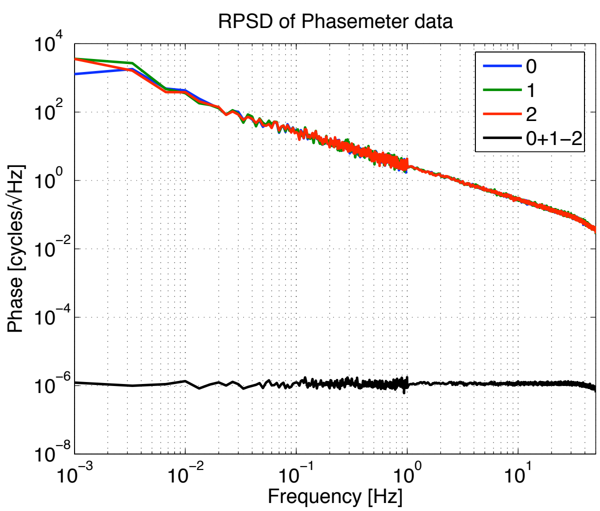


Fig. 7. Root power spectral density of the resultant shown in Figure 6.

signal generator, and the phase error of the difference is due to ADC jitter. This can be removed if necessary by modulating the clock tone onto the signal and subtracting it out.

Another requirement is that the phasemeter be able to track the heterodyne frequency as it varies with the orbit. We have measured the tracking rate of the phasemeter to be 1 MHz/s.

V. Conclusion

A phase measurement system has been developed which is capable of achieving the accuracy requirements for a future Earth gravity mapping mission and for a future space gravitational-wave observatory. The phase measurement system is about two orders of magnitude more accurate than the measurement system used on the GRACE mission while being able to cope with dynamic range variations more than four orders of magnitude larger. The system can be implemented in a flight system by porting the software to a dedicated FPGA combined with a high-speed digitizer. The measurement system also performs the functions needed for locking one

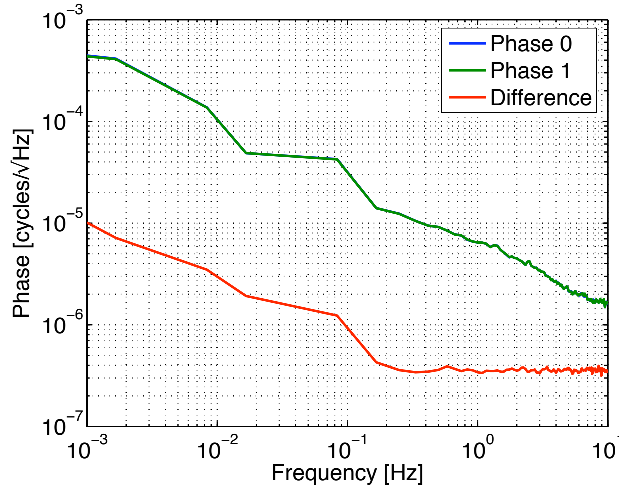


Fig. 8. Measurement of the same input on two different channels, and the difference between the two inputs. The noise on Phase 0 and Phase 1 is due the signal generator; the noise of the difference is due to the ADC jitter.

laser frequency to a reference. A breadboard measurement system has been built and tested to demonstrate the required performance. A copy of this phasemeter is being tested for a spacecraft-spacecraft interferometric range transceiver for future Earth gravity mapping mission [3].

Acknowledgment

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